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Linear inequalities, Miscellaneous

Q.12 A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 L of the 8% solution, how many litres of the 2% solution will have to be added?

Soln: Let the 2% boric acid solution be x litre.
 $\therefore$ Mixture $= (640+x)$ L

Now, according to the question, here, two conditions arise.

I $2\% \text{ of } x + 8\% \text{ of } 640 > 4\% \text{ of } (640+x)$

II $2\% \text{ of } x + 8\% \text{ of } 640 < 6\% \text{ of } (640+x)$

From condition I,

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 > \frac{4}{100} \times (640+x)$$

$$\Rightarrow 100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640 \right] > \frac{4}{100} \times (640+x) \times 100$$

$$\Rightarrow 2x + 8 \times 640 > 4 \times 640 + 4x$$

$$\Rightarrow 2x - 4x > 4 \times 640 - 8 \times 640$$

$$\Rightarrow -2x > 640(4-8) \Rightarrow -2x > 640 \times (-4)$$

$$\Rightarrow \frac{-2x}{2} > -2 \times 640 \Rightarrow x < 2 \times 640 \Rightarrow x < 1280 \quad \text{--- (i)}$$

From condition II

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 < \frac{6}{100} \times (640+x)$$

$$\Rightarrow 100 \times \left[\frac{2x}{100} + \frac{8}{100} \times 640 \right] < \frac{6}{100} \times (640+x) \times 100$$

$$\Rightarrow 2x + 8 \times 640 < 6 \times 640 + 6x \Rightarrow 2x - 6x < 6 \times 640 - 8 \times 640$$

$$\Rightarrow -4x < 640(6-8) \Rightarrow -4x < -2 \times 640$$

$$\Rightarrow \frac{-4x}{-4} > \frac{-2 \times 640}{-4} \Rightarrow x > 320 \quad \text{--- (ii)}$$

Hence, from eqns (i) and (ii),

$$320 < x < 1280$$

$$\text{i.e. } x \in (320, 1280)$$

Ans

Q.13. How many litres of water will have to be added to 1125 L of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Soln Let x litre water be added to 1125 L of the 45% solution of acid.

Now, according to the question,

$$25\% \text{ of } (1125 + x) < 45\% \text{ of } 1125 < 30\% \text{ of } (1125 + x)$$

$$\Rightarrow \frac{25}{100} \times (1125 + x) < \frac{45}{100} \times 1125 < \frac{30}{100} \times (1125 + x)$$

$$\Rightarrow 25 \times 1125 + 25x < 45 \times 1125 < 30 \times 1125 + 30x$$

Taking first two parts of above inequality,

$$25 \times 1125 + 25x < 45 \times 1125$$

$$\Rightarrow 25x < 45 \times 1125 - 25 \times 1125$$

$$\Rightarrow 25x < 1125(45 - 25) \Rightarrow 25x < 1125 \times 20$$

$$\Rightarrow \frac{25x}{25} < \frac{1125 \times 20}{25} \Rightarrow x < 225 \times 4 \Rightarrow x < 900 \quad \text{--- (i)}$$

Now, taking last two parts of the above inequality,

$$\Rightarrow 45 \times 1125 < 30 \times 1125 + 30x$$

$$\Rightarrow 45 \times 1125 - 30 \times 1125 < 30x$$

$$\Rightarrow 1125(45 - 30) < 30x \Rightarrow 30x > 1125 \times 15$$

$$\Rightarrow 1125 \times 15 < 30x \Rightarrow 30x > 1125 \times 15$$

$$\Rightarrow \frac{30x}{30} > \frac{1125 \times 15}{30} \Rightarrow x > \frac{1125 \times 15}{2} \Rightarrow x > 562.5 \quad \text{--- (ii)}$$

Hence, from eqns (i) and (ii), we get

$$562.5 < x < 900 \text{ i.e. } x \in (562.5, 900).$$

The number of litres of water to be added should be greater than 562.5 L and less than 900 L.